NETWORKS WITH VARIABLE RESISTANCE

BY

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E. L. NELSON

ARMOUR INSTITUTE OF TECHNOLOGY

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NETWORKS WITH VARIABLE RESISTANCE.

A THESIS

PRESENTED BY

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NETWORKS WITH VARIABLE RESISTANCE.

Briefly, the problem of the telephone engineer is:- first, to convert sound vibrations in the air to pulsating electrical currents; second, to transmit these currents to a distance; third, to reconvert the electrical impulses received at the distant station into sound; and fourth, to provide suitable means for signaling and for tariff collection.

In all modern telephone systems, the conversion of sound waves into electrical vibrations is accomplished by the use of some form of microphone transmitter. This device is essentially a resistance which is caused to vary under the action of the sound waves and thereby produce a corresponding variation of the current in the circuit in which it is placed. It generally takes the form of a loose contact between carbon elements, one of which is connected to a diaphragm upon which the sound waves

impinge.

The transmission of the telephonic currents to moderate distances does not present any unusual difficulties. In early systems, a single wire with a ground return was used to connect the stations and proved satisfactory. As the number of subscribers in a given system increased, however, various modifications were required to prevent cross-talk, to promote efficient operation, and to improve the service. This development has brought about a greater complication of the system, and the modern telephone circuit is a rather intricate network containing various resistances, inductances, and capacities.

The reconversion of the currents into sound at the receiver is accomplished by the action of their magnetic field on a thin, soft iron diaphragm. A permanently magnetized core carrying a coil of fine wire through which the currents pass is placed near the latter and the varying field thus produced causes it to vibrate and

thereby emit sound. This is the active principle of the original Bell receiver which has been used up to the present time with but little modification.

The problem of signaling and tariff collection, although supplementary to that of speech transmission, is usually important from a commercial and operating standpoint. Many of the complications of the modern telephone system have been introduced merely to gain some slight advantage along these lines. The matter, therefore requires careful consideration and is intimately related to the problem of speech transmission inasmuch as the same conductors must be used in both cases.

The work of this thesis is confined to a study of the talking circuits, i.e., those which contain the transmitting and receiving devices and in which the telephonic currents flow. The object has been to determine the effect upon the current of viaration of the various quantities resistance, inductance, and capacity in the cir-

cuit. The investigation has been conducted along both theoretical and experimental lines, and includes a study of a number of the circuits in use by some of the leading telephone manufacturers in the country.

The General Problem.

In general, the telephone circuit is a network containing various electromotive forces, inductances, capacities, and resistances, of which one of the latter (the transmitter) is variable. The solution of the general problem for such a circuit has recently been accomplished by Professor H. W. Nichols of the Armour Institute of Technology. The following is a brief abstract from a paper prepared by him on this subject:

NETWORKS WITH VARIABLE RESISTANCE.*

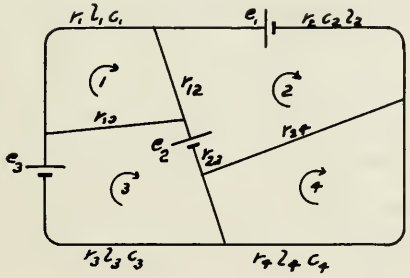
Take a network whose branches contain resistance, inductance, capacity, and in which

*Abstract by Prof. H.W.Nichols.

different branches may be related by mutual inductance and capacity. Let one of the meshes, say (1), contain a resistance variable with the time and of the form:

$$r_1 = r_o (1 - aP)$$

where "a" is the amplitude of the pulsation of resistance and "P" is a periodic function of amplitude unity and frequency $\frac{p}{2\pi}$. Batteries, or other sources of constant electromotive force, are supposed to be in any of the meshes, and it is required to find the current in any mesh when the resistance "r_1" is varied in the manner expressed in the equation above.



Let "x_k" be any mesh current, "$\dot{x}_k$" its time rate of change, and "$\int x_k$" its time integral; then for any mesh in the network, by the generalized Kirchoff Laws:

$$R_K x_K + L_K \dot{x}_K + \frac{1}{C_K} + M_j \dot{x}_j \cdots\cdots\cdots = \Sigma e_K$$

There will be as many of these equations as there are independent mesh currents, say:

$$r_{11} x_1 + l_{11} \dot{x}_1 + \frac{1}{C_K} x_K + m_{12} \dot{x}_2 + m_{13} \dot{x}_3 + \cdots\cdots\cdots$$
$$- r_{12} x_2 - l_{12} \dot{x}_2 \cdots = e_1$$

$$- r_{21} x_1 + r_{22} x_2 + \cdots\cdots l_{22} \dot{x}_2 + \frac{1}{C_{22}} \dot{x}_2 + \cdots\cdots\cdots$$
$$- l_{21} \dot{x}_1 - \cdots\cdots = e_2$$

$$- r_{31} x_1 - r_{32} x_2 + r_{33} x_3 + l_{33} \dot{x}_3 + \frac{1}{C_{33}} x_3 + \cdots\cdots\cdots$$
$$- l_{31} \dot{x}_1 + \cdots\cdots = e_3$$

To solve this system of equations, put

$$x = X + \xi \qquad \dot{x} = \iota \rho x$$

where "x" is the constant current which would flow if the amplitude of the pulsations were zero; it therefore obeys the laws of distribution of direct currents in a conducting network. "ξ" is the (small) variation in this steady current which is impressed due to the

variation in the resistance in the mesh (1).

Then the rate of change of "x" is "$\dot{\xi}$", since "x" is constant, and the current "x" is zero for any mesh which contains a condenser. If we make this substitution in the system of equations above, they become:

$$r_0(1+aP)(x_1+\xi_1) + \iota p l_1 \dot{\xi} - \frac{\iota}{p C_1}\xi_1 + m_{12}\iota p \xi_2 + \cdots\cdots$$
$$- r_{12}(x_2+\xi_2) - r_{13}(x_3+\xi_3) - \cdots = e_1$$

or:

$$r_0(1-aP)\xi_1 + \iota p l_1 \dot{\xi}_1 + \cdots\cdots - r_{12}\xi_2 - r_{13}\xi_3 - \cdots\cdots$$
$$= e_1 + aP r_0 x_1 + r_{12}x_2 + r_{13}x_3 \cdots\cdots$$

Now since "a" is small omit it in the left hand member, and introduce the impedances, "z_k", giving:

$$Z_{11}\xi_1 - Z_{12}\xi_2 - Z_{13}\xi_3 + \cdots\cdots = e_1 + aP r_0 x_1 + r_{12}x_2 + \cdots$$
$$-Z_{21}\xi_1 + Z_{22}\xi_2 - Z_{23}\xi_3 + \cdots\cdots = e_2 + \cdots\cdots$$
$$-Z_{31}\xi_1 - Z_{32}\xi_2 + Z_{33}\xi_3 - \cdots\cdots = e_3 + \cdots\cdots$$

where

$$Z_{11} = r_{11} + \iota\left(p l_{11} - \frac{1}{C_{11}p}\right)$$
$$Z_{12} = r_{12} + \iota p l_{12} - \frac{\iota}{p C_{12}} + \iota p m_{12} + \cdots$$

By further transformation, it is found that every term on the right which does not contain "P" will vanish leaving as the final form of the

system of equations:

$$Z_{11}\xi_1 - Z_{12}\xi_2 - Z_{13}\xi_3 - \cdots\cdots = ar_0 Px_1$$
$$-Z_{21}\xi_1 + Z_{22}\xi_2 - Z_{23}\xi_3 - \cdots\cdots = 0$$
$$\cdots\cdots = 0$$

These show that the system of conductors may be considered to be excited by a single alternator of electromotive force "ar_0Px_1", so that the problem is reduced to one in alternating currents. The solution is, if "D" represents the determinant of the coefficients, and "D_{jk}" the minor of the j^{th} row and the k^{th} column:

$$\xi_K = ar_0 Px_1 \frac{D_{jk}}{D}$$

Now the steady currents "x" satisfy the same equations as the alternating currents when we omit the terms containing the imaginary. Hence:

$$x_1 = \begin{vmatrix} e_1 - r_{12} & -r_{13} & \vdots \\ e_2 + r_{22} & -r_{23} & \vdots \\ e_3 - r_{32} & +r_{33} & \vdots \\ \cdots\cdots & \cdots\cdots & \cdots\cdots \\ r_{11} & -r_{12} & \cdot \\ -r_{21} & -r_{22} & \cdot \\ -r_{31} & -r_{32} & \cdot \end{vmatrix} = \frac{\Delta_1}{\Delta}$$

and the final form of the solution for any "ξ" is:

$$\mathcal{E}_K = a r_0 P \frac{\Delta_1}{\Delta} \frac{D_{jK}}{D}.$$

This equation is perfectly general except inasmuch as the variation of resistance is assumed to follow a sine law. It may be applied therefore, to any network for the calculation of the maximum value of the current at any frequency.

Experimental Method.

The general procedure followed in this work has consisted in setting up the circuit to be studied, measuring the receiver current for different values of the variables, and comparing the results thus obtained with those found by calculation.

As has been shown above, the general equation expressing the current in the receiver in terms of the constants of the circuit is:

$$\xi = a r_o \frac{\Delta_1}{\Delta} \frac{D_1}{D},$$

where "a" is the amplitude of the pulsation of resistance caused by the vibration of the transmitter diaphragm, "r_o" the average resistance of the transmitter, "P" a periodic function of amplitude unity, and "Δ_1", "Δ", "D", and "D_1" determinants composed of the resistances, reactances, and electromotive forces of the various meshes. The experimental work, therefore, in-

volves the use of a transmitter, for which the amplitude and frequency of the pulsations of resistance, as well as the average resistance are known, and the measurement of the receiver current and of the various resistances, inductances, capacities, and electromotive forces in the circuit.

As no data is available on the percent variation in resistance of a microphone, it was necessary to use some other deivce for exciting the circuits. The arrangement devised for this purpose consisted of a non-inductive resistance of the ordinary plug form, a portion of which was periodically short-circuited by means of an electrically driven tuning fork fitted with contacts. The connections were such that 20 ohms out of a total of 110 were short-circuited. The amplitude of the pulsations of resistance was, therefore, 0.1 and the average resistance 100 ohms. The frequency of the pulsations, as determined by that of the fork, was 257.

For the measurement of the receiver current, a sensitive galvanometer and a thermo-couple was used. The latter consisted of a heating element similar in construction to an ordinary incandescent lamp but having a straight filament. The thermo-couple proper was connected to the center of the heater and the connecting wires sealed through the glass stem of the bulb in the usual manner. The bulb was inclosed in a wooden box and insulated against currents of air and against shocks by several layers of felt, the connections being brought out to suitable binding posts on the outside. The galvanometer was a Leeds and Northrup instrument of the moving coil type, the deflections of which were observed by means of a telescope and scale.

The instrument was calibrated on direct current. The scheme of connections used for this purpose was as shown in Fig. 1. The heater was connected in series with a variable

resistance and a single cell of storage battery and the galvanometer joined direct to the thermocouple. Various currents were allowed to pass through the heating element and the corresponding deflections of the galvanometer noted. The currents were measured by means of a known resistance and a potentiometer. From the data thus obtained, the calibration curve was plotted with deflections (cm) as abscissae and currents (milli-amperes) as ordinates.

The inductances used in the various circuits were of two types: one, a simple coil with an open magnetic circuit, and the other, a repeat coil with a closed magnetic circuit and primary and secondary windings.

Two coils of the first type were used, each consisting of several layers of insulated wire wound on a solid core. The measurement of their inductance was made by means of the Anderson method, exciting the bridge at the frequency used in the tests by means of the

tuning fork. In this work, the usual trouble was experienced in obtaining sensitive balances due to the presence of iron in the magnetic circuit, but by using a vibration galvanometer tuned to the frequency of the fork for the final adjustments, results were obtained with an accuracy of 5%, which was considered to be sufficient. The data for both coils is recorded in Table #2 which follows.

The repeat coil used was a standard Western Electric Co. coil consisting of a ring shaped core of fine iron wire, upon which were wound two primary and two secondary windings. As the two parts of each winding were always used in series, however, only the total primary and secondary and mutual inductances were measured. The self inductance was measured by means of the Anderson method as before, while, for the mutual inductance, the Carey-Foster Method was used. The data is recorded in Table #2.

The capacities were two standard mica

condensers subdivided into 0.05, 0.05, 0.2, 0.2, and 0.5 micro-farad units, and also two large paper insulated condensers of 24 micro-far ads each divided into 2 micro-farads units. The calibration of these units was considered to be sufficiently accurate for the present purpose and no capacity measurements were made.

The resistances were of the ordinary plug form resistance boxes of the laboratory. These are constructed of insulated wire, doubled back upon itself and wound on spools. The inductance is thus so reduced as to be practically negligible. The calibration of these boxes was found to be within the probable limits of error of the experiment, and they were used without correction.

The source of electromotive force was the large storage battery of the laboratory, connection to which was made by means of a long twin cable. By connecting in more or fewer cells it was possible to obtain a considerable variation

in voltage, but, in general, an electromotive force of about 20 volts was used. The voltage was measured by means of a standard Weston voltmeter, the calibration of which was known. The resistance of the battery and cable was investigated and found to be negligible.

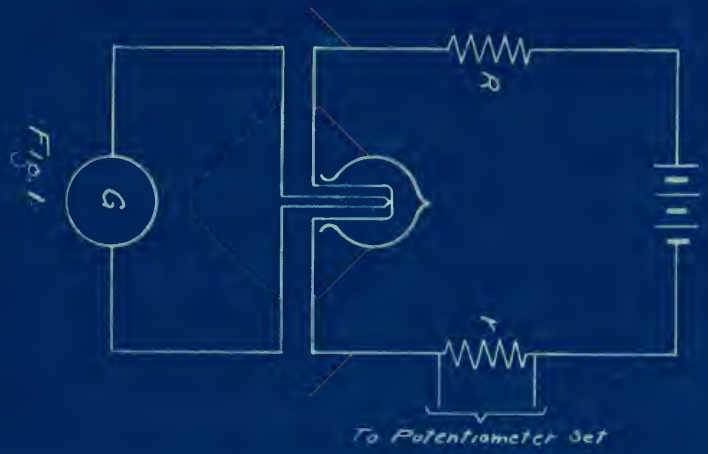


Fig. 1

TABLE #1.

CALIBRATION OF THERMO-COUPLE.

E.M.F. Volts.	R. Ohms.	I. Milli-amps.	Scale Readings. Cm.
0.3555	1000	0.3555	0.525
0.5733	1000	0.5733	1.300
0.8252	1000	0.8252	2.630
1.0485	1000	1.0485	4.210
0.6259	500	1.2518	6.000
0.7416	500	1.4832	8.210
0.8244	500	1.6445	10.100
0.9050	500	1.8100	11.950
1.0120	500	2.0240	14.720
1.0969	500	2.1938	16.920
1.1629	500	2.3258	19.080
5.1370	500	2.5685	22.980
5.4980	500	2.7490	25.770
5.6290	500	2.8145	27.010

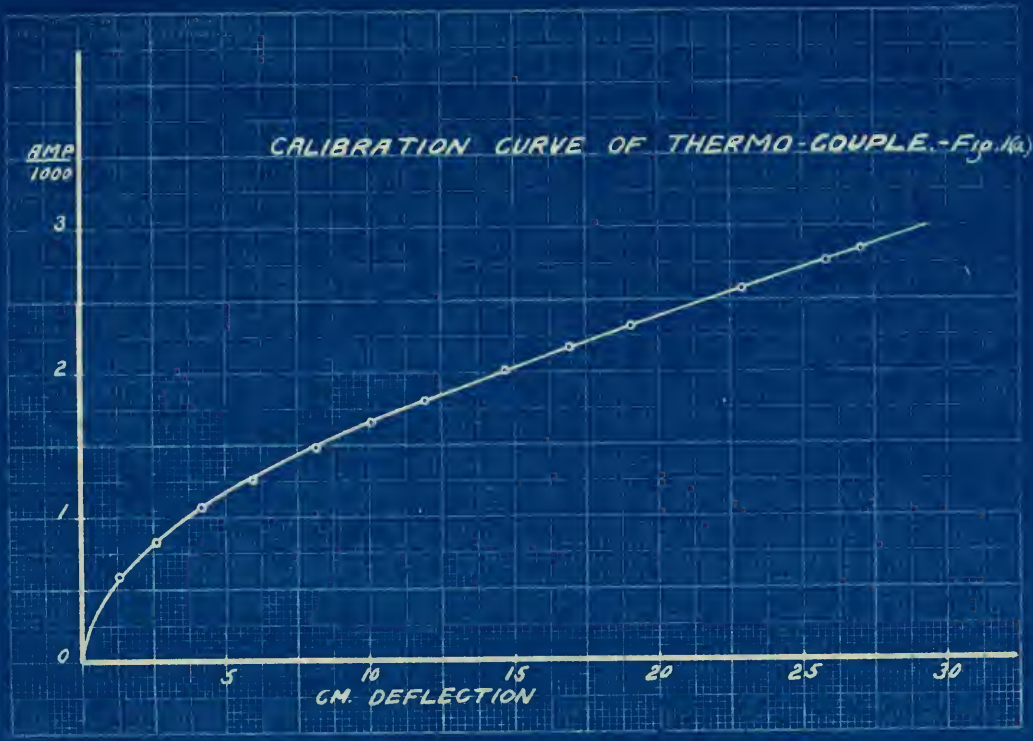

CALIBRATION CURVE OF THERMO-COUPLE.-Fig.1(a)
AMP/1000
3
2
1
0
5
10
15
20
25
30
CM. DEFLECTION

TABLE #2.

MEASUREMENT OF INDUCTANCES.

$$L = \frac{C\,S}{10^6}\left(r\left(\frac{P}{Q}+1\right)+P\right)$$

Coil No.	Q. Ohms	S, Ohms	P. Ohms	r. Ohms	C. micro-farads	L. Henries
120	1300	130	285.9	500	1.5	0.174
	900	9000	2.7	19	0.9	0.176
	1600	160	280.4	250	2.0	0.184
	1550	155	284.2	100	2.83	0.183
	1300	130	272.9	350	2.0	0.181
					Mean - -	0.179
119	3000	300	280.	100	1.5	0.175
	5000	500	280.4	550	0.4	0.192
	8000	800	281.4	206	0.4	0.179
	1250	125	284.2	382	2.0	0.187
	1300	130	285.3	372	1.0	0.185
					Mean - -	0.183

Resistance of coil #120 = 27.5 ohms.

Resistance of coil #119 = 27.4 ohms.

TABLE #2 (Cont'd).

Standard Western Electric Repeat Coil.

Primary Windings.(Tot.Res.=45.2 ohms)

Q. Ohms.	S. Ohms.	P. Ohms.	r. Ohms.	C. micro-farads.	L. Henries
1750	175	453.9	200	1.0	0.122
1700	170	454.0	275	0.9	0.123
1750	175	456.0	180	1.0	0.120
1700	170	456.9	700	0.5	0.114
1800	180	457.5	500	0.6	0.118
				Mean - - - -	0.119

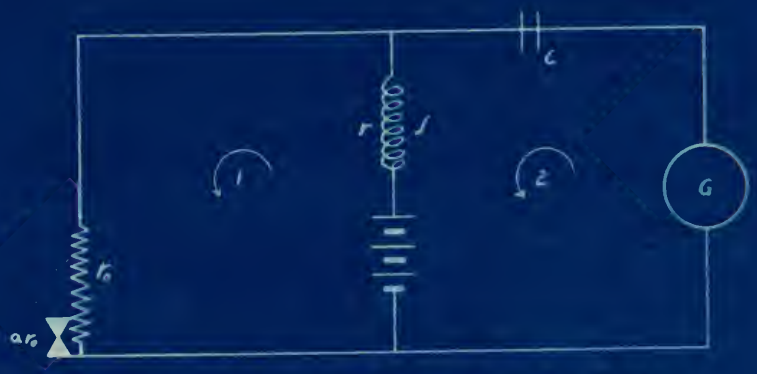
Secondary Windings.(Tot.Res. = 41.1 ohms)

1750	175	416.5	600	0.7	0.142
1750	175	416.5	900	0.5	0.134
1700	170	417.1	950	0.5	0.135
1800	180	417.2	900	0.5	0.139
1800	180	417.3	557	0.7	0.138
				Mean - - - -	0.138

Mutual Inductance.

$$M = \frac{C}{10^6} r R.$$

R. Ohms.	r. Ohms.	C. micro-farads.	M. Henries.
8350	7	2.00	0.117
13900	3	2.83	0.118
18000	7	1.00	0.126
5370	12	2.00	0.129
10400	12	1.00	0.125
		Mean - - - - -	0.123



SIMPLE COMMON BATTERY CIRCUIT - Fig 2

Simple Common Battery Circuit.

The first circuit studied was perhaps the simplest possible form on common battery circuit, the arrangement being as shown in Fig. 2. In this case, there were but two meshes, one containing the transmitter and the other the receiver. The battery was placed in the common side of the two meshes and an inductance was placed in series with it to prevent the telephonic currents from flowing across this shorter path instead of through the receiver.

The development of the equation for this special case is as follows:

Let: a = the amplitude of pulsation of resistance.

r_o = the average resistance of the transmitter.

P = a periodic function of amplitude unity.

x_1 = the steady current in the first mesh.

e = the e.m.f. of the battery.

r = the resistance of the impedance coil.

l = the inductance of the impedance coil.

p = the angular velocity = 2π times the frequency

C = the capacity in the 2nd mesh(farads).

and g = the resistance of the thermo-couple.

Then as shown in the theoretical discussion above, the mesh equations are:

$$\mathcal{E}_1(r_0 + r + jpl) - \mathcal{E}_2(r + jpl) = ar_0 Px_1$$
$$-\mathcal{E}_1(r + jpl) + \mathcal{E}_2\left(r + g + jpl - \frac{1}{jpC}\right) = 0$$

Solving these equations for the receiver current "$\mathcal{E}_2$" gives:

$$\mathcal{E}_2 = ar_0 Px_1 \frac{(r + jpl)}{\begin{vmatrix} r_0 + r + jpl, & -(r + jpl) \\ -(r + jpl), & r + g + jpl - \frac{1}{pC} \end{vmatrix}}$$

Now "x_1" which is the current in the first mesh, may be expressed as:

$$x_1 = \frac{e}{r + r_0}$$

Hence, making this substitution and dividing by $\sqrt{2}$ to change to effective amperes:

and: $$\mathcal{E}_2 = \frac{a r_0 e}{\sqrt{2}(r+r_0)} \quad \frac{r+jp^2}{D}$$

$$D = r(r_0+\rho) + r_0\rho + \frac{2}{C_1} + j\left(\rho^2\left[r_0+\rho\right] - \frac{1}{\rho C_1}\left[r_0+r\right]\right)$$

This is perhaps the most convenient form for calculations.

Data was taken on the receiver current for the following cases:

1. Variation of the receiver current with capacity; no inductance. In this case no impedance coil was used but a non-inductive resistance of 54.9 ohms was inserted in series with the battery. The capacity in the second mesh was varied by suitable steps and the corresponding receiver current noted. The receiver current for each variation of capacity was also determined by calculation and curves plotted with capacities (micro-farads) as abscissae and current (milli-amperes) as ordinates. These are shown in Fig. 3.

2. Variation of the receiver current with capacity; small inductance. In this case, one

of the inductance coils and a non-inductive resistance of 27.4 ohms was inserted in series with the battery and the capacity varied as before. The observed and the calculated curves thus obtained are shown in Fig. 4.

3. Variation of the receiver current with capacity; large inductance. This variation is similar to the above except that both of the inductance coils were connected in series with the battery. The curves are shown in Fig. 5.

4. Variation of the receiver current with resistance. This test was conducted with one impedance coil and a capacity of 2 micro-farads in circuit. A non-inductive resistance was connected in series with the battery and the receiver current noted for the different values of the latter. The curves obtained in this manner and by calculation are shown in Fig. 6.

TABLE #3

SIMPLE COMMON BATTERY CIRCUIT.

Variation of Current with Capacity.

No Inductance.

e = 20.9 volts. l = 0 henries.
g = 315 ohms. r = 54.9 ohms.
r_o = 100 ohms. a = 0.1
p = 1680.

C. Micro-farads.	I. Milli-amperes.	$\mathcal{E}_2$ Milli-amperes.
0.25	0.53	0.14
0.50	0.62	0.27
1.00	0.73	0.49
4.00	0.88	0.57
10.00	0.90	0.95
48.00	0.92	0.96

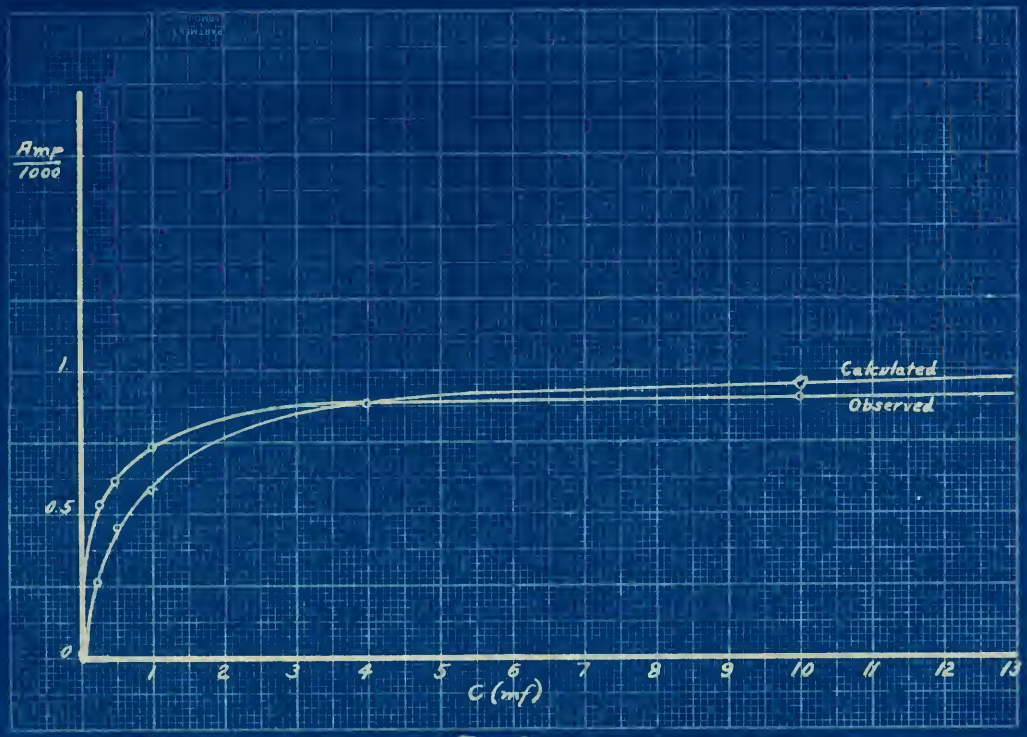


Fig 3

TABLE #4.

SIMPLE COMMON BATTERY CIRCUIT.

Variation of Current with Capacity.

Small Inductance.

e = 20.9 volts. l =0.179 henries.
g = 315 ohms. r =54.9 ohms.
r_o = 100 ohms. a =0.1
p = 1680.

C. Micro-farads.	I. Milli-amperes.	$\mathcal{E}_2$ Milli-amperes.
0.25	1.28	0.36
0.50	1.48	0.70
0.66	1.60	0.90
1.00	1.79	1.18
1.50	1.86	1.58
2.00	1.96	1.78
4.00	2.02	2.06
10.00	2.09	2.14
48.00	2.09	2.15

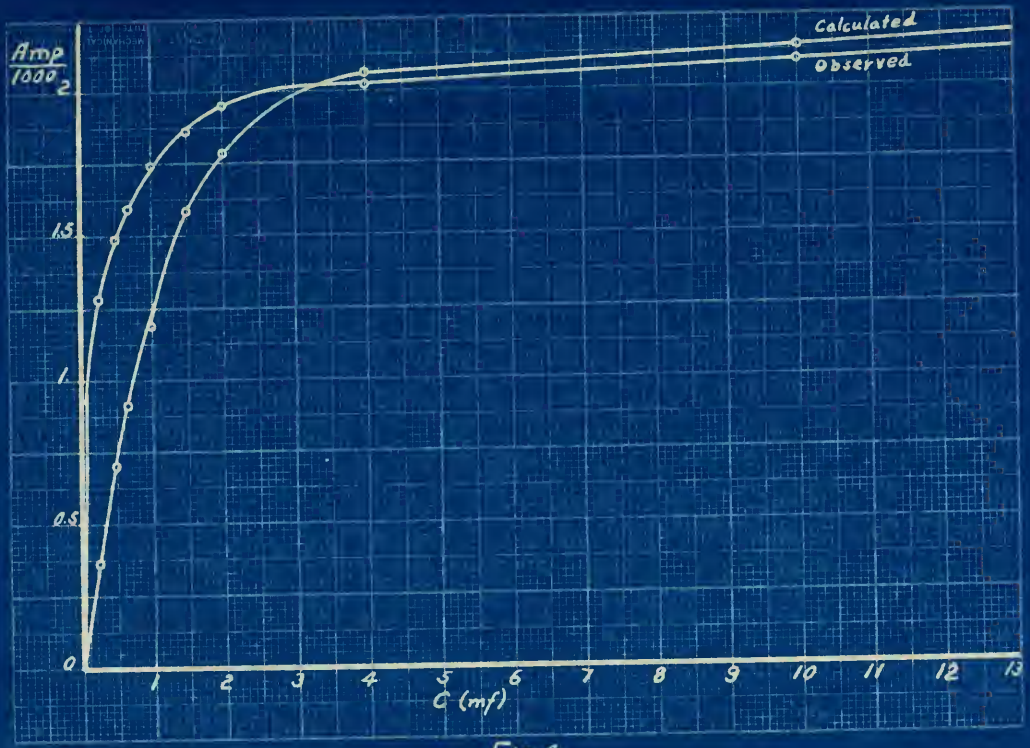

Amp
1000
2
1.5
1
0.5
0
1
2
3
4
5
6
7
8
9
10
11
12
13
C (mf)
Calculated
Observed

Fig. 4

TABLE #5.

SIMPLE COMMON BATTERY CIRCUIT.

Variation of Current with Capacity.

Large Inductance.

e = 20.9 volts. l = 0.362 henries.
g = 315 ohms. r = 54.9 ohms.
r_o = 100 ohms. a = 0.1
p = 1680.

C. Micro-farads.	I. Milli-amperes.	$\mathcal{E}_2$ Milli-amperes.
0.25	1.28	0.384
0.50	1.53	0.726
0.66	1.61	0.955
1.00	1.77	1.310
1.50	1.92	1.650
2.00	1.97	1.865
4.00	2.06	2.100
10.00	2.10	2.240
48.00	2.10	2.260

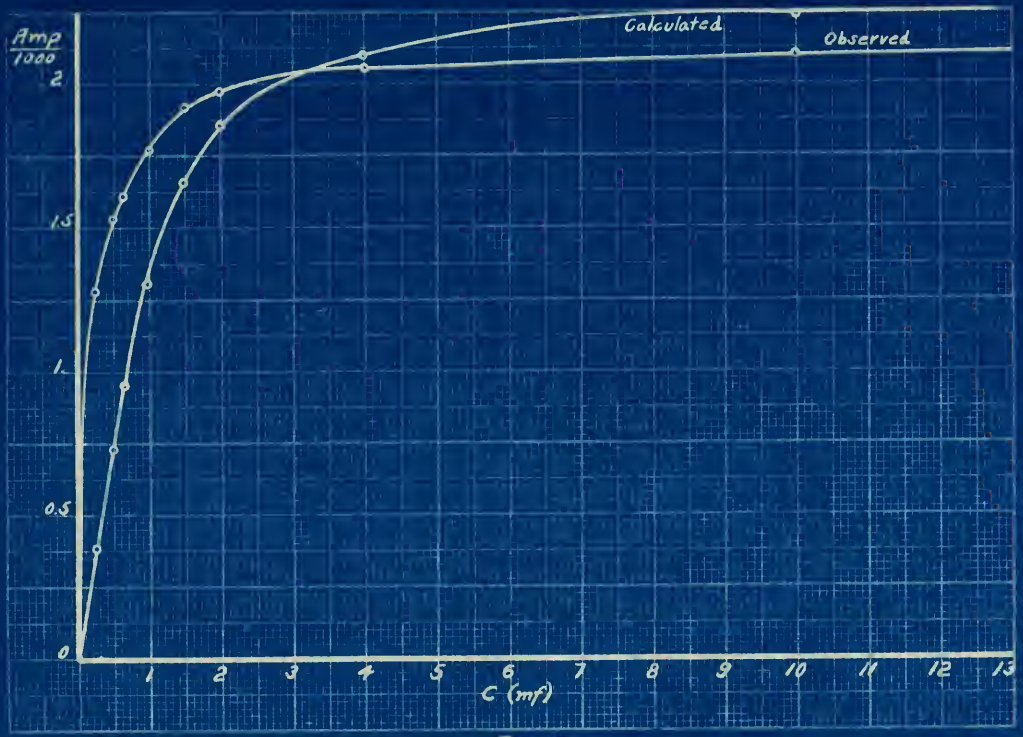

Amp/1000
2
1.5
1
0.5
0
Calculated
Observed
1
2
3
4
5
6
7
8
9
10
11
12
13
C (mf)

Fig. 5.

TABLE #6.

SIMPLE COMMON BATTERY CIRCUIT.

Variation of Current with Resistance.

e = 20.9 volts. l = 0.179 henries.
g = 315 ohms. C = 2.0 micro-farads.
r_o = 100 ohms. a = 0.1
p = 1680.

r. Ohms.	I. Milli-amperes.	f_2 Milli-amperes.
27.4	2.40	2.22
37.4	2.24	2.04
47.4	2.10	1.79
57.4	1.91	1.75
67.4	1.81	1.63
77.4	1.71	1.52
102.4	1.51	1.38
127.4	1.36	1.15
177.4	1.12	0.94
227.4	0.94	0.73
327.4	0.72	0.53
527.4	0.54	0.35
1027.4	0.35	0.11

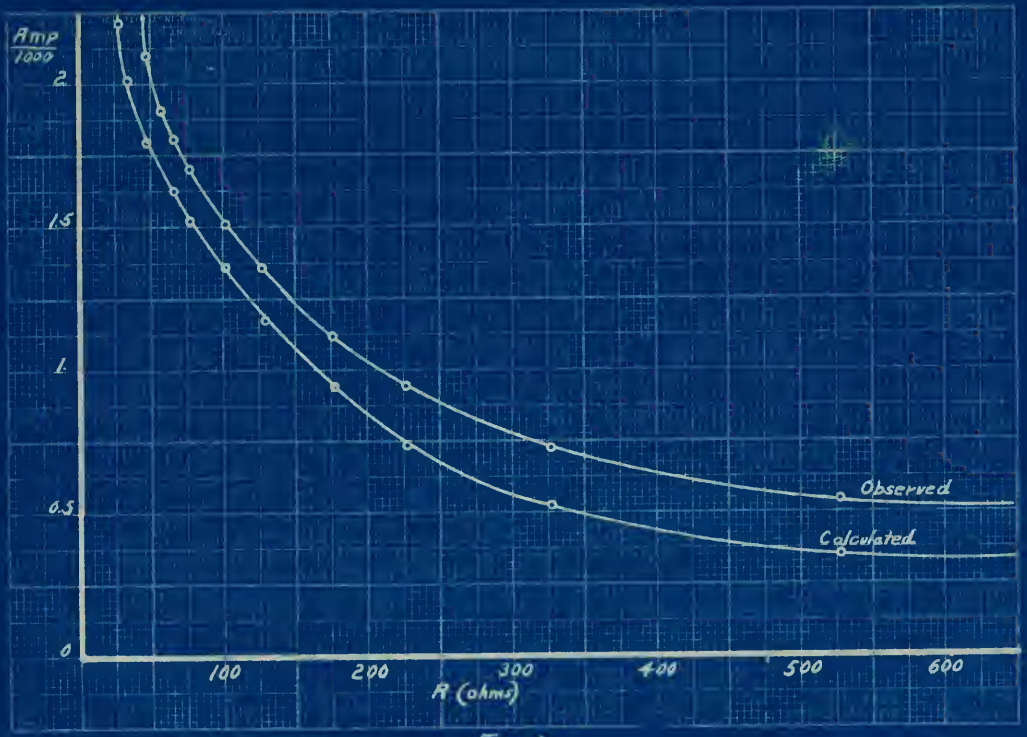
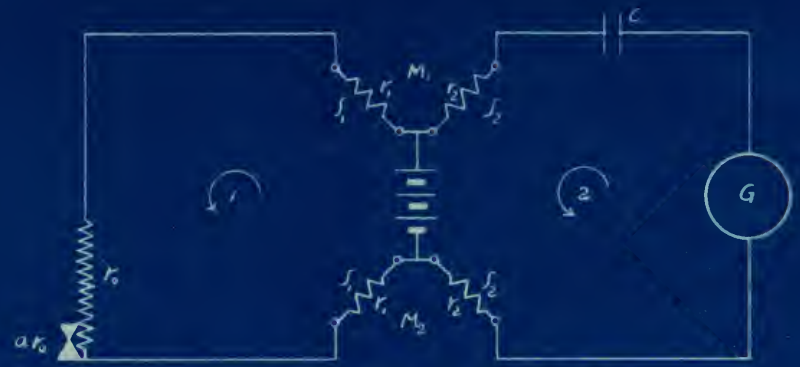

Amp
1000
2
1.5
1
0.5
0
100
200
300
400
500
600
R (ohms)
Observed
Calculated

Fig 6.



WESTERN ELECTRIC CIRCUIT - Fig 7

The Western Electric Circuit.

The circuit used by the Western Electric Company consists of two meshes connected by mutual inductance as shown in Fig. 7.The windings l_1 and l_2 are the primaries and secondaries, respectively, of an induction coil with a closed magnetic circuit. To obtain a symmetrical arrangement these windings are split in the center and the battery inserted at this point. In the test of this circuit, the coil used was one of the standard type made by this company for their commercial installations.

The development of the formula for the calculation of the receiver current is as follows:

Let: a-= the amplitude of pulsation of resistance.

r_o= the average resistance of the transmitter.

P = a periodic function of amplitude unity.

x_1= the steady current in the first mesh.

L_1= 2 l_1= total primary self-inductance.

L_2= 2 l_2= total secondary self-inductance.

$M = M_1 M_2$ = total mutual inductance.

$R_1 = 2r_1$ = the total primary resistance.

$R_2 = 2r_2$ = the total secondary resistance.

C = the capacity in the second mesh(farads).

p = the angular velocity = 2π x frequency.

and g = the resistance of the thermo-couple.

Then, the equations for the meshes are:

$$\mathcal{E}_1(r_0+R_1+jpL_1) - \mathcal{E}_2(jpM) = ar_0 P x_1$$

$$-\mathcal{E}_1 jpM + \mathcal{E}_2\left(R_2+g+jpL_2 - j\frac{1}{pC}\right) = 0$$

Solving for the receiver current "$\mathcal{E}_2$" gives:

$$\mathcal{E}_2 = ar_0 P x_1 \frac{jpM}{\begin{vmatrix} r_0+R_1+jL_2, & -jpM \\ -jpM, & R_2+g+jpL_2-j\frac{1}{pC} \end{vmatrix}}$$

But "x_1" the current in the first mesh, is:

$$x_1 = \frac{e}{r_0+R_1}$$

Hence, making this substitution and dividing by $\sqrt{2}$ to obtain the effective amperes:

$$\mathcal{E}_2' = \frac{ar_0}{\sqrt{2}} \frac{e}{r_0+R_1} \frac{jpM}{D}$$

and "D" = $(r_0+R_1)(R_2+g)+\frac{L_1}{C} - p^2(L_1L_2-M^2)$

$$+j\left\{pL_1(R_2+g)+pL_2(r_0+R_1)-\frac{1}{pC}(r_0+R_1)\right.$$

Experimentally, it was not possible to

obtain a large number of variations with this circuit because the self and mutual inductances were necessarily fixed. Data was taken, however, on the following:

1. Variation of the receiver current with capacity. The procedure in this case consisted simply in varying the capacity in the second mesh and observing the corresponding currents. The current was also determined by calculation and curves plotted with capacity (micro-farads) as abscissae and current (milli-amperes) as ordinates. (Fig. 8).

2. Variation of the receiver current with resistance. A non-inductive resistance was connected in series with the battery and readings of the current taken with this as the variable. This data and a similar set obtained by calculation were plotted in the form of the curves shown in Fig. 9.

3. Variation of the receiver current with the amplitude of pulsation of resistance. This

portion of the test was undertaken primarily to obtain some indication as to the action of the tuning fork. The portion of the transmitter resistance which was short-circuited was varied through as wide a range as possible, keeping the average resistance of the device constant. Data was taken on the receiver current and curves were plotted with values of "a" as abscissae and current (milli-amperes) as ordinates. These are shown in Fig. 10.

TABLE #7

WESTERN ELECTRIC CIRCUIT.

Variation of Current with Capacity.

e = 20.7 volts. L_1 = 0.119 henries.
r_o = 100 ohms. L_2 = 0.138 henries.
g = 315 ohms. M = 0.123 henries.
p = 1680 R_1 = 45.2 ohms.
a = 0.1 R_2 = 41.1 ohms.

C. Micro-farads.	I. Milli-amperes.	$\mathcal{E}_2'$ Milli-amperes.
0.25	0.78	0.17
0.50	1.16	0.48
0.66	1.31	0.70
1.00	1.59	1.24
1.50	1.84	1.53
2.00	1.98	1.68
2.50	2.06	1.75
3.00	2.10	1.80
3.50	2.14	1.82
5.00	2.17	1.83
10.00	2.18	1.84
24.00	2.20	1.84
48.00	2.21	1.84

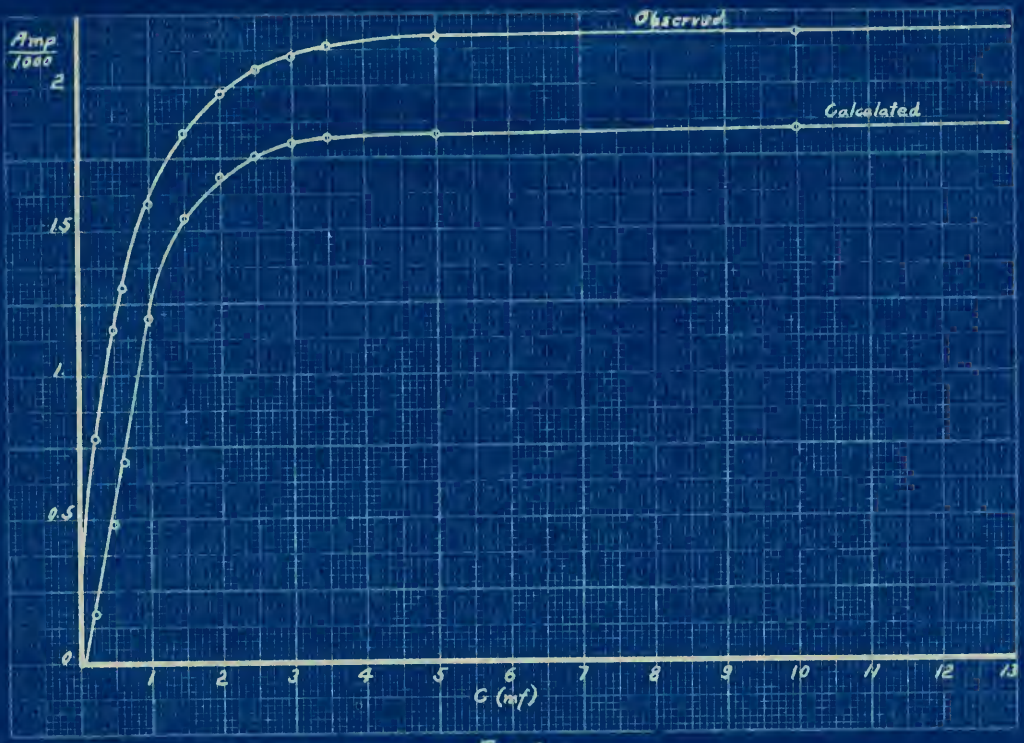

Amp
1000
2
1.5
1
0.5
0
Observed
Calculated
1
2
3
4
5
6
7
8
9
10
11
12
13
C (mf)

Fig 8

TABLE #8.

WESTERN ELECTRIC CIRCUIT.

Variation of Current with Resistance.

e = 21.2 volts. L_1 = 0.119 henries.
r_o = 100 ohms. L_2 = 0.138 henries.
g = 315 ohms. M = 0.123 henries.
p = 1680. R_1 = 45.2 ohms.
a = 0.1 R_2 = 41.1 ohms.
C = 2.0 micro-farads.

R. Ohms.	I. Milli-amperes.	$\mathcal{E}_2'$ Milli-amperes.
0	1.98	1.73
20	1.71	1.38
60	1.30	1.10
125	1.03	0.57
200	0.82	0.34
300	0.62	0.18
500	0.46	0.07

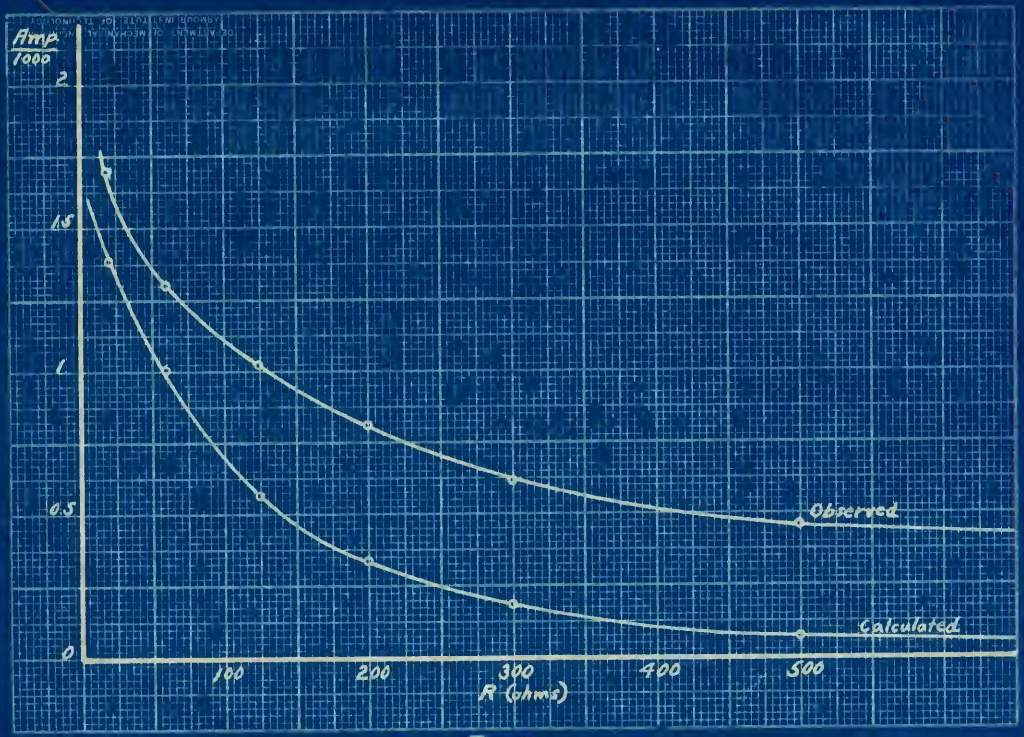

Amp
1000
2
1.5
1
0.5
0
100
200
300
400
500
R (ohms)
Observed
Calculated

Fig 9

TABLE #9.

WESTERN ELECTRIC CIRCUIT.

Variation of Current with the Amplitude

of the Pulsation of Resistance.

e = 21.2 volts. L_1 = 0.119 henries.
r_o = 100 ohms. L_2 = 0.138 henries.
g = 315 ohms. M = 0.123 henries.
p = 1680 R_1 = 45.2 ohms.
C = 2.0 mic.-farads. R_2 = 41.1 ohms.

a.	I. Milli-amperes.	$\mathcal{E}_2'$ Milli-amperes.
0.015	0.18	0.26
0.025	0.48	0.43
0.035	0.66	0.60
0.050	0.91	0.86
0.060	1.15	1.04
0.070	1.36	1.21
0.080	1.62	1.38
0.090	1.86	1.56
0.100	2.09	1.73

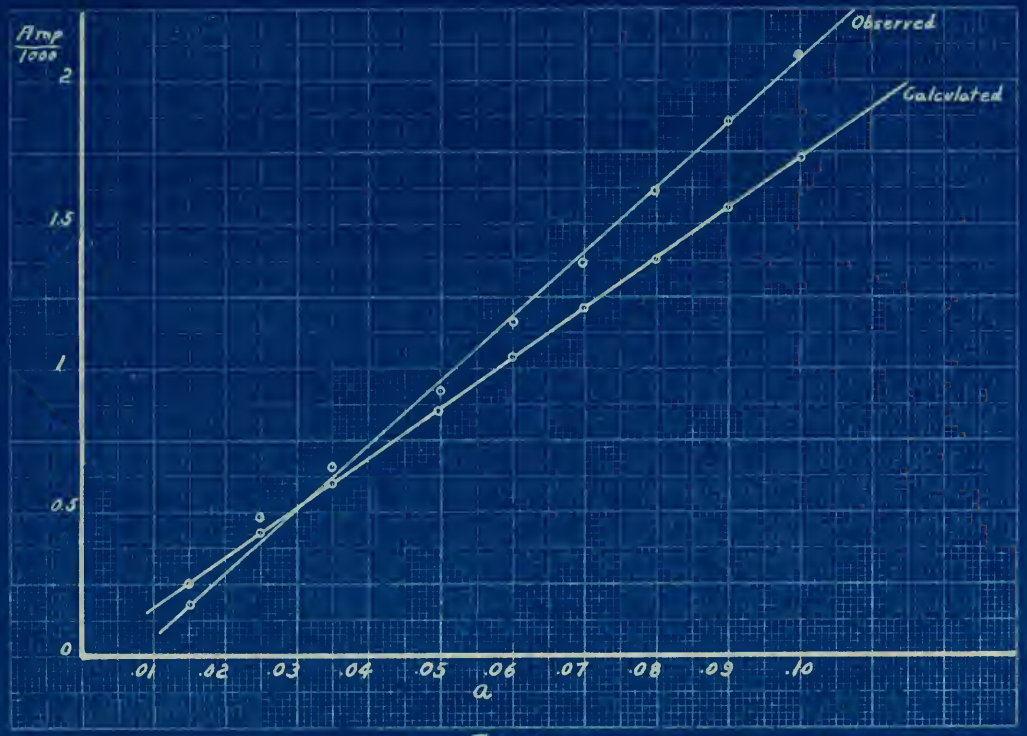
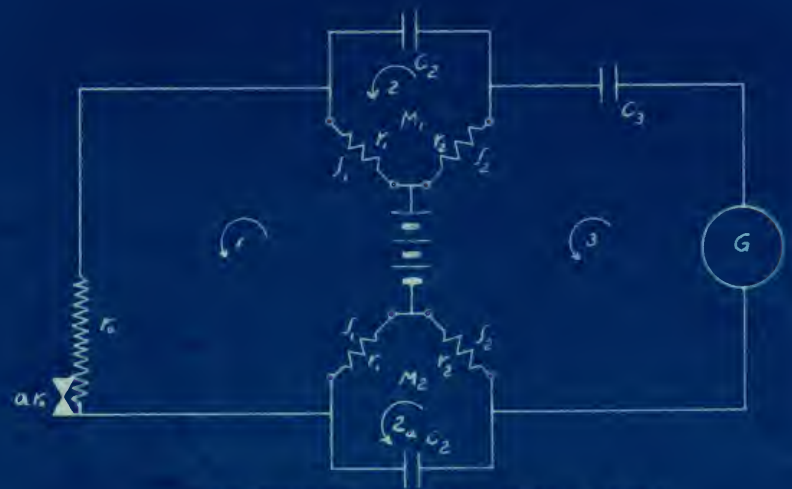

Amp/1000
2
1.5
1
0.5
0
Observed
Calculated
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
a

Fig 10



STROMBERG-CARLSON CIRCUIT - Fig 11

The Stromberg-Carlson Circuit.

The Stromberg-Carlson circuit resembles the preceding one inasmuch as the transmitter and receiver meshes are connected by mutual induction between two symmetrically placed primaries and secondaries. This circuit differs, however, in that capacities are connected between the outer terminals of the primary and secondary windings, forming a network of four meshes.

Since there are four meshes, the formula for the calculation of the receiver current becomes somewhat complicated and difficult to use. As the arrangement is perfectly symmetrical about a horizontal line drawn through its center we may consider it to be divided along this line and calculate the current in one of these networks of three meshes thus formed, if the constants are taken for each of the meshes as one half of those for the entire mesh. Then,

if the electromotive force acting is also divided by two, the current thus calculated will be the true receiver current. The equation for this method of calculation may be developed as follows:

Let: a = the amplitude of pulsation of resistance.

r_o = the average resistance of the transmitter.

p = a periodic function of amplitude unity.

x_1 = the steady current in the first mesh.

p = the angular velocity = 2π x frequency.

and let the other constants in the circuit be represented as indicated in Fig. 11.

Then:

$$Z_{11} = \frac{r_o}{2} + r_1 + jp l_1$$
$$Z_{12} = r_1 + jp l_1$$
$$Z_{22} = r_1 + r_2 + jp(l_1 + l_2) - j\frac{1}{pC_2}$$
$$Z_{23} = r_2 + jp l_2$$
$$Z_{33} = r_2 + \frac{1}{2}g + jp l_2 + j\frac{2}{pC_3}$$

The mesh equations are therefore:

$$\xi_1 Z_{11} - \xi_2 Z_{12} + jpM_1 \xi_3 = \frac{a r_o p x_1}{2}$$
$$-\xi_1 Z_{12} + \xi_2 Z_{22} - Z_{23}\xi_3 = 0$$
$$jpM_1 \xi_1 - \xi_2 Z_{23} + \xi_3 Z_{33} = 0$$

Solving for the receiver current "ξ_3":

$$\xi_3 = a r_0 P x_1 \quad \frac{Z_{12} Z_{23} - jpM_1 Z_{22}}{\begin{vmatrix} Z_{11} & -Z_{12} & jpM_1 \\ -Z_{12} & Z_{22} & -Z_{23} \\ jpM_1 & -Z_{23} & Z_{33} \end{vmatrix}}$$

But "x_1", the current in the first mesh, is:

$$x_1 = \frac{e}{r_0 + 2r_1}$$

Hence, making this substitution and dividing by $\sqrt{2}$ to obtain the effective current:

$$\xi_3' = \frac{e}{(r_0+2r_1)2\sqrt{2}} \frac{a r_0 \quad Z_{12} Z_{23} - jpM_1 Z_{22}}{\left[Z_{33}(Z_{11}Z_{22} - Z_{12}^2) + \left\{Z_{23}(-Z_{11}Z_{23} + jpM_1 Z_{12}) + jpM_1(Z_{12}Z_{23} - jpM_1 Z_{22})\right\}\right.}$$

This is the most convenient form for calculations.

The variations studied for this circuit are as follows: Only observed curves are plotted.

1. Variation of the receiver current with capacity. The capacity in the third mesh was varied and the receiver current observed. The data is plotted in the form of the curve shown in Fig. 12, where the abscissae are the total capacity in the third mesh and the ordinates are the receiver currents in milli-amperes.

2. Variation of the receiver current with

capacity. In this case the capacities C_2 and C_2' connected across the induction coils were varied throught as wide a range as possible but keeping them always equal. The corresponding variation in the current is shown in Fig. 13 where the abscissae represent the capacity in each condenser.

3. Variation of the current with resistance. A non-inductive resistance was inserted in series with the battery and varied through as wide a range as possible. The curve thus obtained is shown in Fig. 14.

4. Variation of the receiver current with the amplitude of pulsation of resistance. The procedure in this case consisted in varying the portion of the transmitter resistance short-circuited while maintaining the average resistance constant. The results were plotted as shown in Fig. 15, with values of "a" as abscissae and the corresponding current (milli-amperes) as ordinates.

TABLE #10.

STROMBERG-CARLSON CIRCUIT.

Variation of Current with Capacity.

e = 21.3 volts.	L_1 = 0.119 henries.
r_o = 100 ohms.	L_2 = 0.138 henries.
g = 315 ohms.	M = 0.123 henries.
p = 1680	R_1 = 45.2 ohms.
a = 0.1	R_2 = 41.1 ohms.

C_2 = 2 micro-farads.

C_3 Micro-farads.	I. Milli-amperes.
0.25	0.95
0.50	1.25
0.66	1.44
1.00	1.70
1.50	1.98
2.00	2.04
2.50	2.13
3.00	2.29
3.50	2.33
5.00	2.38
15.00	2.40
24.00	2.41

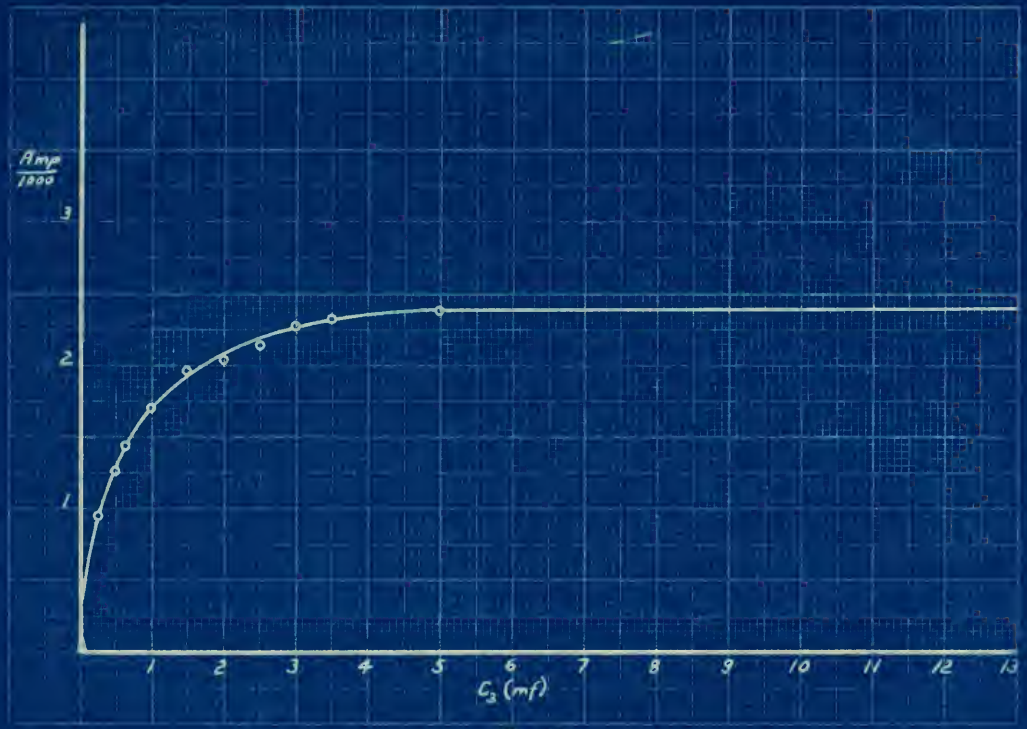


Fig 12

TABLE #11.

STROMBERG-CARLSON CIRCUIT.

Variation of Current with Capacity.

e = 21.3 volts.	L_1 = 0.119 henries.
r_o = 100 ohms.	L_2 = 0.138 henries.
g = 315 ohms.	M = 0.123 henries.
p = 1680	R_1 = 45.2 ohms.
a = 0.1	R_2 = 41.1 ohms.

C_3 = 2 micro-farads.

C_2 Micro-farads.	I. Milli-amperes.
0.25	2.04
0.50	2.05
1.00	2.09
1.50	2.11
2.00	2.12
4.00	2.26
10.00	2.36
24.00	2.48

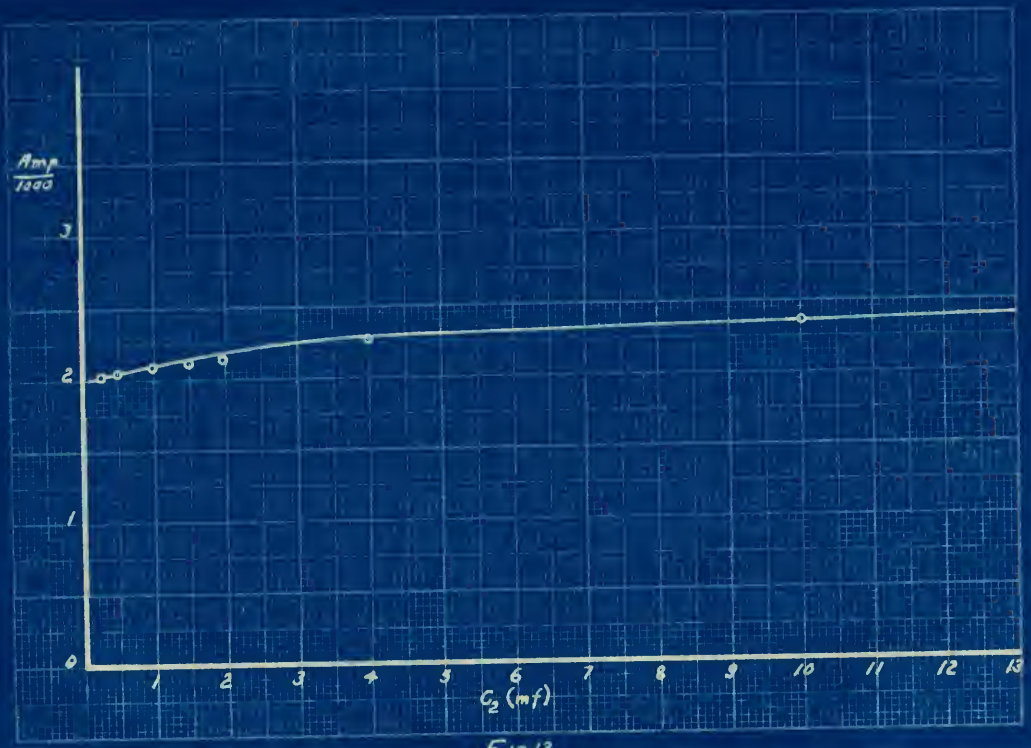


Fig. 13

TABLE #12.

STROMBERG-CARLSON CIRCUIT.

Variation of Current with Resistance.

e = 21.3 volts. L_1 = 0.119 henries.
r_o = 100 ohms. L_2 = 0.138 henries.
g = 315 ohms. M = 0.123 henries.
p = 1680. R_1 = 45.2 ohms.
a = 0.1 R_2 = 41.1 ohms.
C_3 = 2.0 micro-farads. C_2 = 2.0 micro-farads.

R. Ohms.	I. Milli-amperes.
0	2.10
5	2.03
15	1.82
25	1.70
50	1.45
75	1.26
100	1.13
150	0.95
200	0.88
350	0.57
1000	0.24

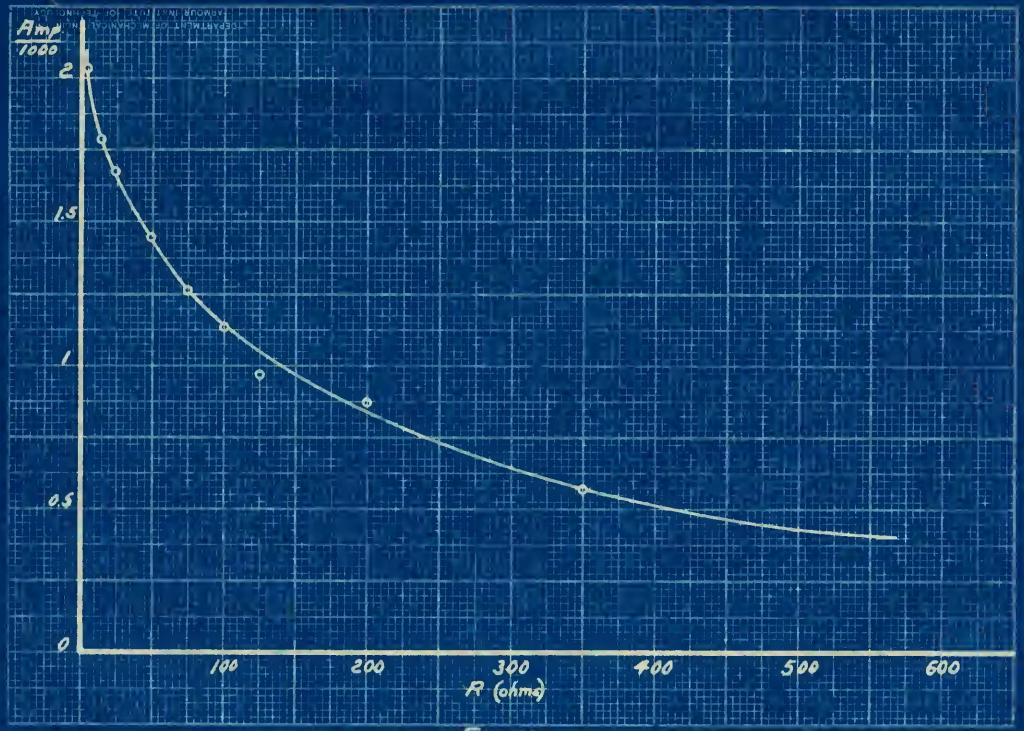


Fig 14

TABLE #13.

STROMBERG-CARLSON CIRCUIT.

Variation of Current with the Amplitude of the Pulsation of Resistance.

e = 21.3 volts. L_1 = 0.119 henries.
r_o = 100 ohms. L_2 = 0.138 henries.
g = 315 ohms. M = 0.123 henries.
p = 1680 R_1 = 45.2 ohms.
C_2 = 2.0 micro-farads. R_2 = 41.1 ohms.
C_3 = 2.0 micro-farads.

a.	I. Milli-amperes.
0.010	0.10
0.015	0.22
0.025	0.48
0.035	0.70
0.050	1.02
0.060	1.24
0.070	1.48
0.080	1.66
0.090	1.88
0.100	2.10
0.111	2.34

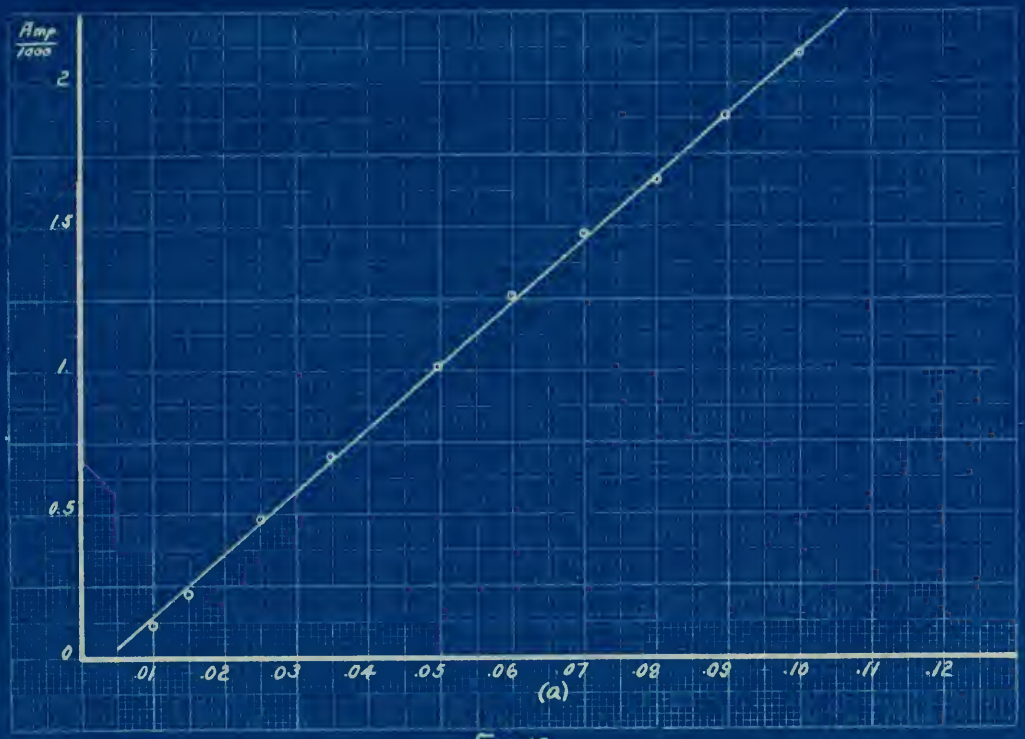
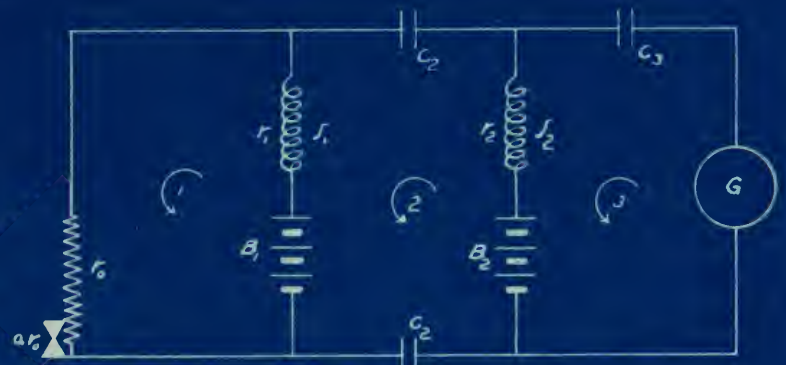


Fig 15



KELLOGG CIRCUIT - Fig 16

The Kellogg Circuit.

The Kellogg circuit consists of three meshes in which two batteries are used as shown in Fig. 16. In the arrangement shown, the battery B_2 is practically on open circuit and has no particular function to perform, but under working conditions, where speech is transmitted in both directions, it supplied current to the second transmitter.

The development of the equation for this circuit is as follows:

Let a = amplitude of the pulsation of resistance.

r_o= average resistance of the transmitter.

P = a periodic function of amplitude unity.

x_1= the steady current in the first mesh.

z_{11}= the "self" impedance of the first mesh.

z_{12}= the mutual impedance of meshes 1 and 2.

z_{22}= the self impedance of the second mesh.

z_{23}= the mutual impedance of meshes 2 and 3.

and z_{33}= the self impedance of the third mesh.

where:

$$Z_{11} = r_0 + r_1 + jp l_1$$

$$Z_{12} = r_1 + jp L_1$$

$$Z_{22} = r_1 + r_2 + jp(l_1 + l_2) - j\frac{1}{2pC_2}$$

$$Z_{23} = r_2 + jp l_2$$

and $Z_{33} = r_2 + \rho + jp l_2 - j\frac{1}{pC_3}$

(Notation as in Fig. 16)

Then, the mesh equations are:

$$\xi_1 Z_{11} - \xi_2 Z_{12} - 0 = a r_0 P x_1$$

$$-\xi_1 Z_{12} + \xi_2 Z_{22} - \xi_3 Z_{23} = 0$$

$$0 - \xi_2 Z_{23} + \xi_3 Z_{33} = 0$$

Solving for the receiver current "ξ_3" gives:

$$\xi_3 = a r_0 P x_1 \frac{Z_{12} Z_{23}}{\begin{vmatrix} Z_{11} & -Z_{12} & 0 \\ -Z_{12} & Z_{22} & -Z_{23} \\ 0 & -Z_{23} & Z_{33} \end{vmatrix}}$$

But x_1, the current in the first mesh, is:

$$x_1 = \frac{e}{r_1 + r_0}$$

Hence, making this substitution and dividing by 2 to obtain the effective amperes:

$$\xi_3' = \frac{a r_0 P x_1 (Z_{12} Z_{23})}{\sqrt{2}\left[Z_{11} Z_{22} Z_{33} - Z_{12}^2 Z_{33} - Z_{23}^2 Z_{11}\right]}$$

A number of variations are possible in this case, of which the following have been studied. The curves shown are all plotted from experimental data, no theoretical curves having been worked out for this circuit.

1. Variation of the receiver current with capacity. In this case the capacity in the third mesh was varied and the receiver current observed. From this data the curve shown in Fig. 17 was plotted, laying off capacities (micro-farads) as abscissae and current (milli-amperes) as ordinates.

2. Variation of the receiver current with capacity. This is a variation of the capacity in the second mesh, maintaining that in the third mesh constant at 2.0 micro-farads. The two capacities C_2 and C_2' were kept equal but varied through as wide a range as possible and the receiver currents noted. The curve thus obtained is shown in Fig. 18 where the abscissae represent the capacity of each condenser and not the total in the mesh.

3. Variation of the receiver current with resistance in series with the battery B_2. In this case a non-inductive resistance was placed in series with the battery B_2 and a series of readings taken with this quantity as a variable. The curve obtained is shown in Fig. 19.

4. Variation of the receiver current with resistance in series with the battery B_1. The procedure in this case is similar to that in the above. The data obtained was plotted in the form of the curve shown in Fig. 20

5. Variation of the receiver current with resistance in series with both batteries. Non-inductive resistances were connected in series with both batteries and readings taken with these as the variables, keeping both resistances always equal. The curve showing the relation between resistance and current under these conditions is shown in Fig. 21.

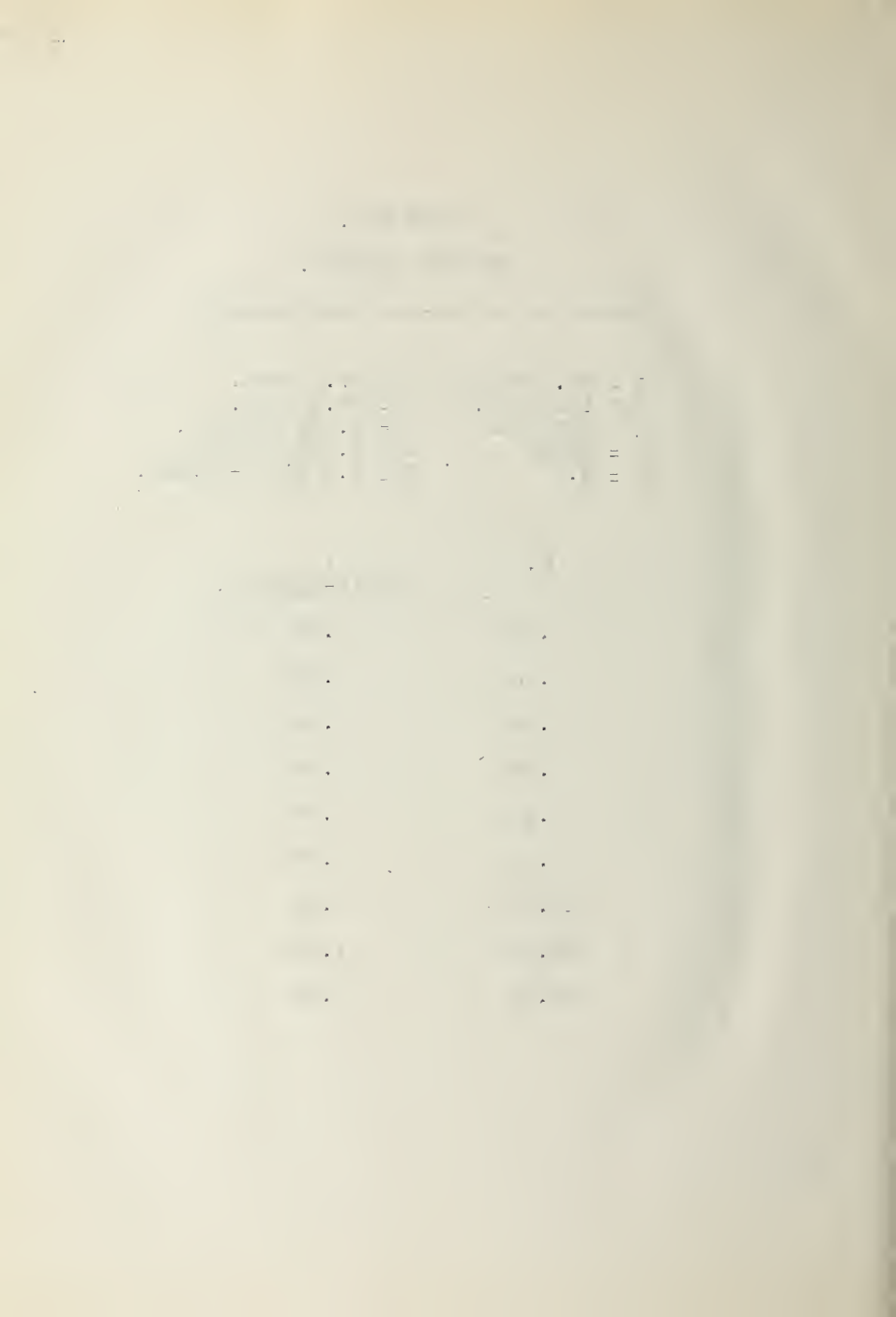
TABLE #14.

KELLOGG CIRCUIT.

Variation of Current with Capacity.

e = 21.2 volts. r_1 = 27.4 ohms.
r_o = 100 ohms. r_2 = 27.5 ohms.
g = 315 ohms. l_1 = 0.179 henries.
p = 1680 l_2 = 0.183 henries.
a = 0.1 C_2 = 2.0 micro-farads.

C_3	I. Milli-amperes.
0.25	1.04
0.50	1.43
0.66	1.54
1.00	1.74
1.50	1.84
4.00	1.92
10.00	1.94
16.00	1.95
24.00	1.95



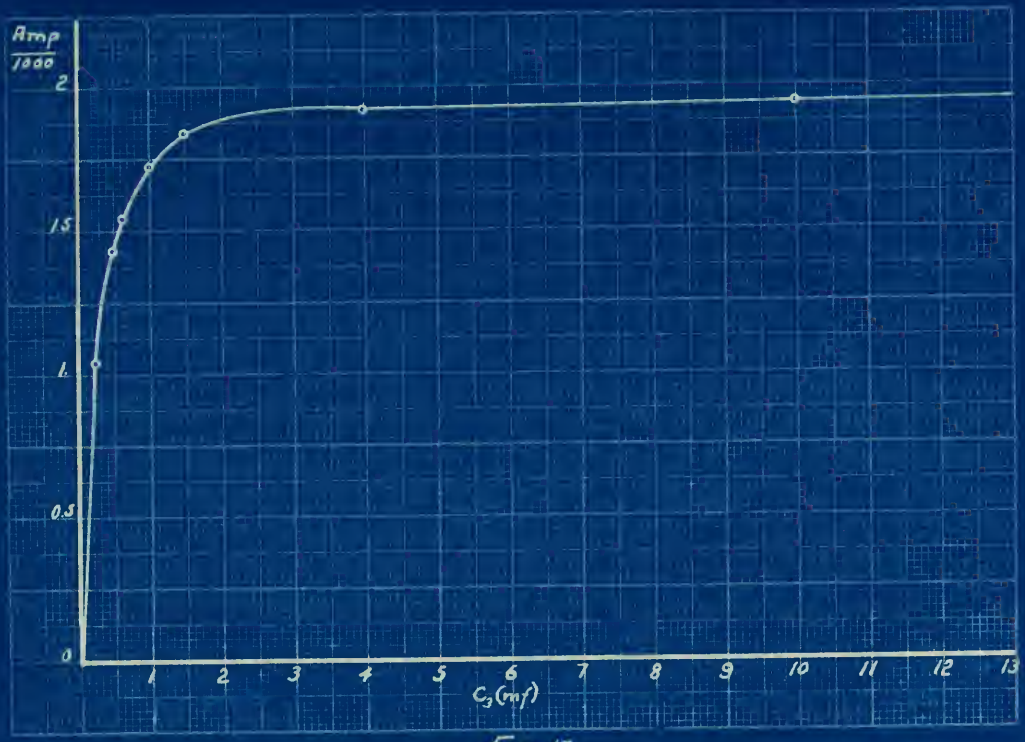


Fig 17

TABLE #15.

KELLOGG CIRCUIT.

Variation of Current with Capacity.

e = 21.2 volts.	r_1 = 27.4 ohms.
r_o = 100 ohms.	r_2 = 27.5 ohms.
g = 315 ohms.	l_1 = 0.179 henries.
p = 1680.	l_2 = 0.183 henries.
a = 0.1	C_3 = 2.0 micro-farads.

C_2. Micro-farads.	I. Milli-amperes.
0.25	0.50
0.50	0.85
0.66	1.03
1.00	1.28
1.50	1.56
2.00	1.79
2.50	1.95
3.00	2.06
4.00	2.20
6.00	2.30
10.00	2.35
16.00	2.38
24.00	2.39

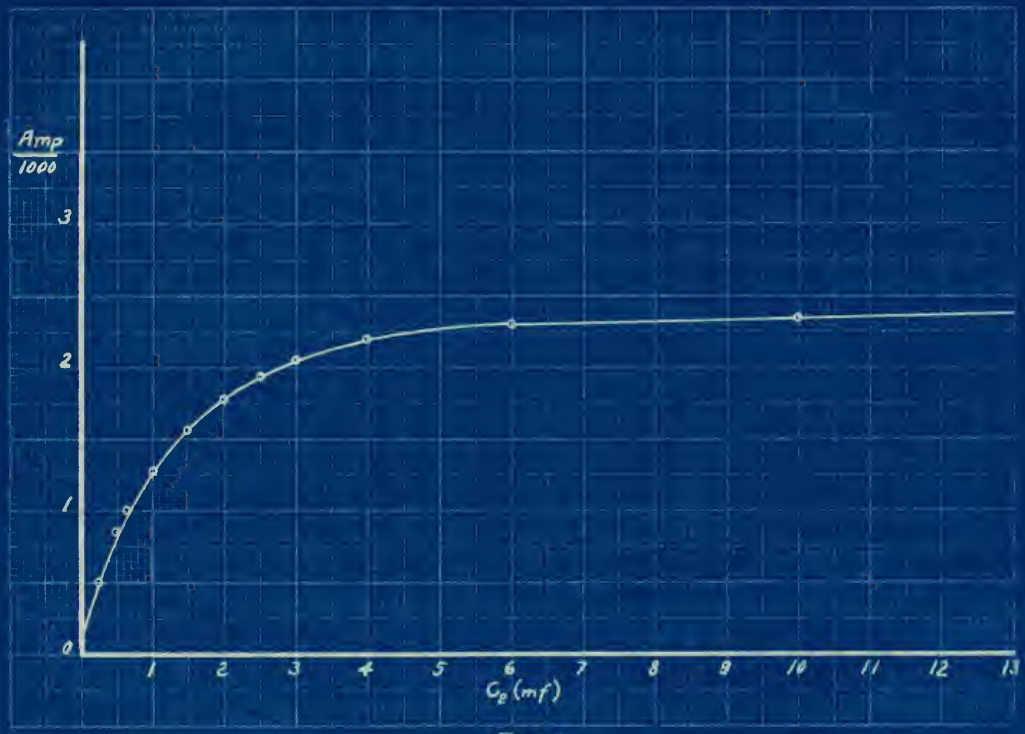


Fig 18

TABLE #16.

KELLOGG CIRCUIT.

Variation of Current with Resistance.

(Resistance in Series with Battery B_1.)

e = 21.2 volts.	r_1 = 27.4 ohms.
r_o = 100 ohms.	r_2 = 27.5 ohms.
g = 315 ohms.	l_1 = 0.179 henries.
p = 1680.	l_2 = 0.183 henries.
a = 0.1	C_3 = 2.0 micro-farads.

C_2 = 2.0 micro-farads.

R. Ohms.	I. Milli-amperes.
0	1.73
5	1.68
10	1.61
25	1.50
50	1.29
100	1.02
500	0.35

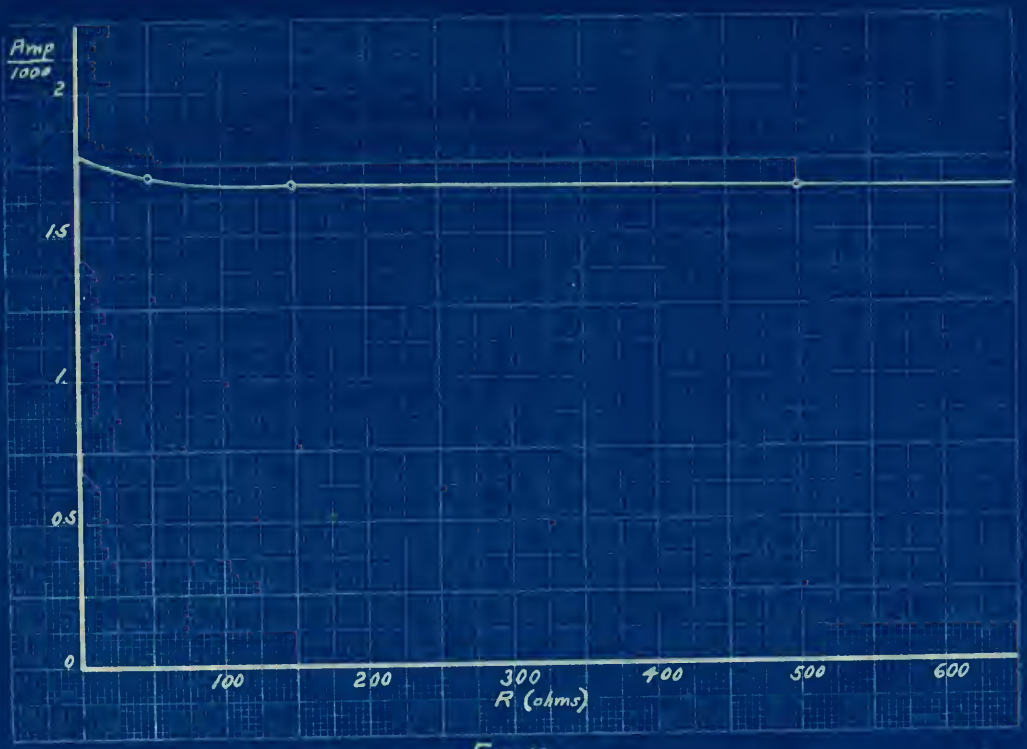


Fig. 19

TABLE #17.

KELLOGG CIRCUIT.

Variation of Current with Resistance.

(Resistance in Series with Battery B_2.)

e = 21.2 volts. r_1= 27.4 ohms.
r_o= 100 ohms. r_2= 27.5 ohms.
g = 315 ohms. l_1= 0.179 henries.
p = 1680 l_2= 0.183 henries.
a = 0.1 C_3= 2.0 micro-farads.

C_2= 2.0 micro-farads.

R. Ohms.	I. Milli-amperes.
0	1.79
50	1.71
100	1.68
500	1.66
5000	1.61

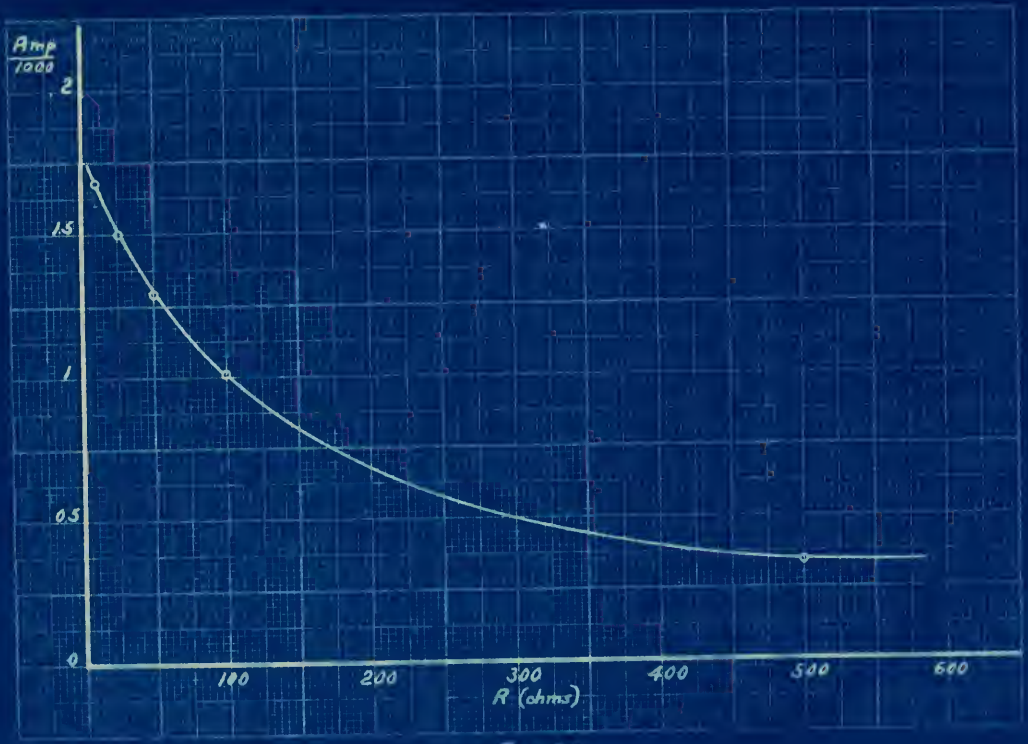


Fig 20

TABLE #18

KELLOGG CIRCUIT.

Variation of Current with Resistance.

(Equal Resistances in Series with Both Batteries.)

e = 21.2 volts. r_1 = 27.4 ohms.
r_o = 100 ohms. r_2 = 27.5 ohms.
g = 315 ohms. l_1 = 0.179 henries.
p = 1680. l_2 = 0.183 henries.
a = 0.1 C_3 = 2.0 micro-farads.

C_2 = 2.0 micro-farads.

R. Ohms.	I. Milli-amperes.
5	1.85
10	1.69
25	1.49
50	1.26
100	0.97
500	0.30

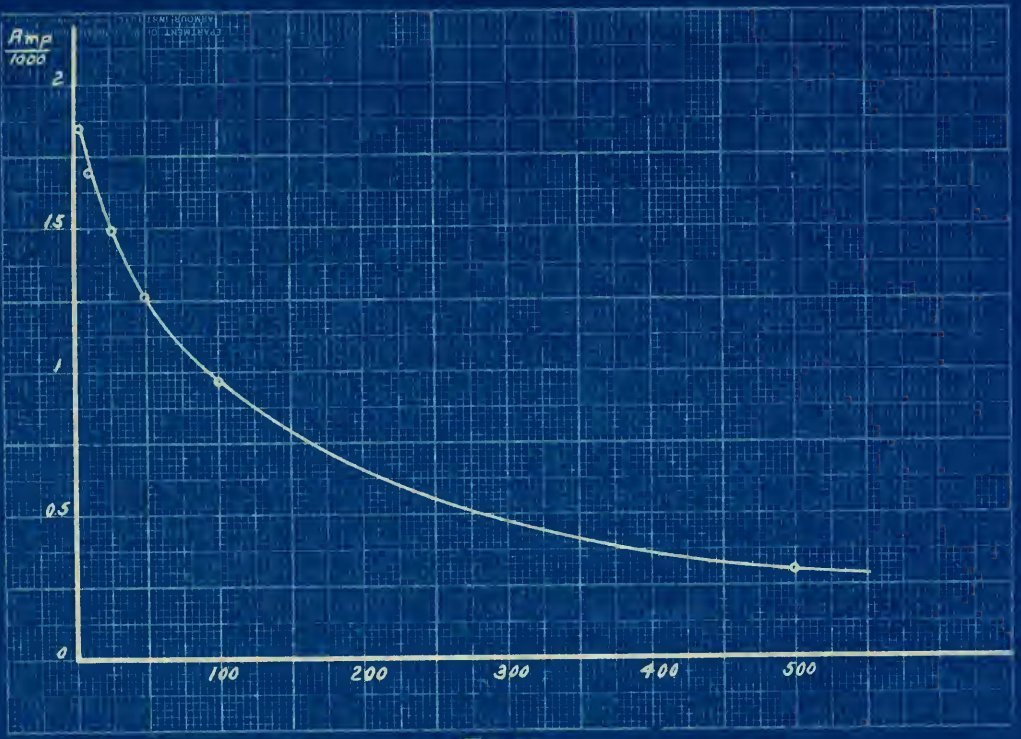


Fig 21

CONCLUSION.

In beginning the work of this thesis it was considered advisable to use as simple a circuit as possible, in order to become familiar with the general principles involved and to perfect the general method. One of the simplest common battery circuits was chosen therefore, such as is shown in Fig. 2. This circuit contains all of the elements of the more complicated types and is thoroughly practical although it is not used commercially in the form illustrated.

The experimental work was laid out very carefully in order to determine whether the type of transmitter used was suitable and also to obtain an insight into the general characteristics of the circuit. The actual procedure followed has already been explained in detail on the preceeding pages and need not be repeated here. There are a number of points about the curves, however, which are worthy of note and which will be mentioned in the following discussion.

The first three curves (Figs. 3, 4, and 5) show the variation of the receiver current with the capacity in the second mesh for different values of inductance. These curves all show a marked similarity in form, the current rising rapidly as the capacity is increased up to three micro-farads, after which additional capacity produces but little change. The agreement between the observed and calculated curves is not especially good for small values of the capacity, but the similarity in general is so marked as to indicate that this departure is due more to error in measuring the constants of the circuit rather than to a difference in the law of variation.

By comparing the three curves, it is also possible to obtain some indication as to how the current in the receiver is affected by the inductance. It will be noted that as the inductance is increased from zero to 0.2 henry, approximately, the current is more than doubled while a

further increase of 0.2 henry produces but little change. It would seem, therefore, that the curve for this variation is similar to those shown for capacity.

The curves of Fig. 6 show the effect upon the current of a variable resistance placed in series with the battery. The result is as would be expected, the receiver current falling rapidly to a low value. The observed and the calculated curves in this case are practically parallel but the former shows considerably higher values of the current for a given value of the resistance. This effect is caused, no doubt, by the presence of harmonics in the current wave due to the characteristics of the transmitter. From the description of the latter given, it is evident that the device will not, under ordinary conditions, produce a pure sine wave. Hence the equation of the current is of the form:

$$I = a_1 \sin pt + a_2 \sin 2pt \text{ ---- } b_1 \cos pt + b_2 \cos 2pt$$

The current as calculated from the formula is

the effective value of the fundamental, or:

$$f_2 = \frac{1}{\sqrt{2}} \sqrt{a_1^2 + b_1^2}$$

The measured current, on the other hand, is the total equivalent heating current, which may be expressed as:

$$I = \frac{1}{\sqrt{2}} \sqrt{\Sigma a^2 + \Sigma b^2}$$

The latter is obviously the larger by an amount depending upon the purity of the current wave. The curves for the Western Electric Company's circuit are shown in Figs. 8, 9, and 10. Of these, the first two are for the variation with capacity and resistance and present no new features. The last, however, (Fig. 10) which is plotted between values of "a" and current, is worthy of note as an indication upon the working of the transmitter. From the general equation it will be seen that this curve should be a straight line passing through the origin. Experimentally this relation was very approximately obtained although the slope of the line is slightly different.

Figs. 12, 13, 14, and 15 show the results for the Stromberg-Carlson circuit. In this case no theoretical curves have been worked out but the effect of varying the different constants may be clearly seen from the observed data. The most interesting curve in this group is Fig. 13, which shows the variation of the current with the capacity C_2 (Fig. 11). As will be seen the magnitude of this capacity has but little effect on the actual receiver current and in this respect the circuit is little better than that of the Western Electric Company to which it reduces when the capacity is zero.

For variations of capacity and resistance the curves for the Kellogg circuit resemble those of the other circuits in general form. In this circuit, as had been mentioned, the function of the additional battery is to supply current to the second microphone when speech is transmitted in both directions. This battery, however, has but little effect on the current in the receiver as is shown in Fig. 19.

Thus far the discussion has been confined to a description of the more important features in the results. In view, however, of the fundamental nature and the practical importance of the theory set forth in this work, it may be of value to point out its significance.

At the present time commercial telephone circuits are developed by "cut and try" methods. The circuit is set up in such shape as the experience and judgment of the experimenter dictates and the various quantities varied until the best results are obtained. Such procedure leaves much to be desired and is most inefficient in time and labor, but up to the present no material has been available from which the required quantities could be calculated. The equations given on the preceeding pages however afford a means of calculating any of the constants of a given circuit if the others are known or can be assumed. It is therefore of fundamental importance from the standpoint of design and should be of great value.